Week 6: Post-Midterm Stuff!

MATH 4A

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6-1.4 Let $C = \begin{bmatrix} -1 & 2 & -2 & 0 \\ 0 & 0 & 3 & -1 \\ 3 & 0 & -1 & 0 \\ -2 & 1 & 0 & -2 \end{bmatrix}$. Find $\det(C)$.

$$\det C = -1 \det \begin{bmatrix} -1 & 2 & -2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix} - 2 \det \begin{bmatrix} -1 & 2 & -2 \\ 0 & 0 & 3 \\ 3 & 0 & -1 \end{bmatrix}$$

$$= -1\left[-3 \det \begin{bmatrix} 2 & -2 \\ 1 & 0 \end{bmatrix} - (-1) \begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix}\right] - 2\left(-3 \det \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}\right)$$

$$= -1\left[-3(2) + [-1+4]\right] - 2\left[-3(-6)\right]$$

$$= -1(-6+3) - 2(18)$$

$$= 3 - 36 = \boxed{-33}$$

6-1.5 Let $M = \begin{bmatrix} -1 & 0 & 0 & -3 & 0 \\ -2 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 & -2 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$. Find $\det(M)$.

$$\det M = -1 \det \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 2 \\ 0 & 0 & 2 & -2 \\ 1 & 1 & 0 & 0 \end{bmatrix} + (-3) \det \begin{bmatrix} -2 & 0 & 1 & 0 \\ 0 & 3 & 0 & 2 \\ 0 & 0 & 0 & -2 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$= -1\left(-1 \det \begin{bmatrix} 3 & 0 & 2 \\ 0 & 2 & -2 \\ 1 & 0 & 0 \end{bmatrix}\right)$$

$$+3 \left(-(-2) \det \begin{pmatrix} -2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 1 & 1 \end{pmatrix}\right)$$

$$= \det \begin{bmatrix} 3 & 0 & 2 \\ 0 & 2 & -2 \\ 1 & 0 & 0 \end{bmatrix} + 6 \det \left(\begin{pmatrix} -2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 1 & 1 \end{pmatrix}\right)$$

$$= 1 \cdot \det \begin{pmatrix} 0 & 2 \\ 2 & -2 \end{pmatrix} + 6 \cdot (-2) \det \begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}$$

$$= -4 + 6 \cdot (-2) \cdot 3$$

$$= -4 - 36 = \boxed{-40.}$$

6-1.10 If $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = -3$, then what's $\det \begin{bmatrix} a-2g & 8b-16h & c-2i \\ d & 8e & f \\ g & 8h & i \end{bmatrix}$?

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \xrightarrow{(\text{col } 2)\cdot 8} \begin{bmatrix} a & 8b & c \\ d & 8e & f \\ g & 8h & i \end{bmatrix} \xrightarrow{R_1 - 2R_3} \begin{bmatrix} a-2g & 8b-16h & c-2i \\ d & 8e & f \\ g & 8h & i \end{bmatrix}$$

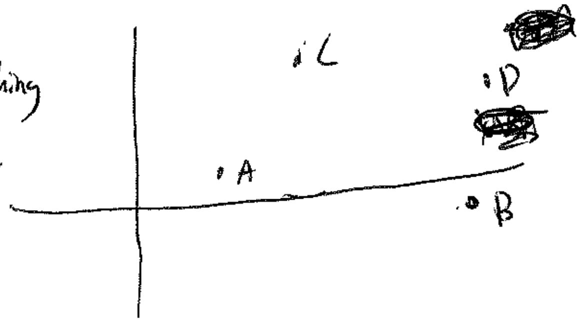
multiplies det. by 8.

leaves ~~the~~ determinant unchanged.

A B C D

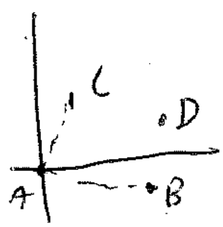
6-1.12 Find the area of the parallelogram with vertices at $(4,1)$, $(16,-2)$, $(6,6)$, and $(18,3)$.

The points look something like:



[not to scale].

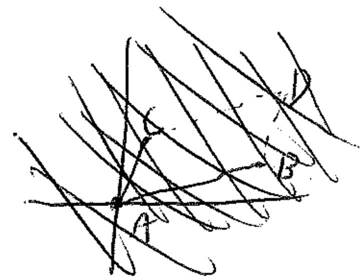
We may move A to $(0,0)$, in which case



$B \rightsquigarrow (12,-3)$

$C \rightsquigarrow (2,5)$

$D \rightsquigarrow (14,2)$



So, the area is given by putting C and B in a matrix and computing abs. value of determinant:

$$\det \begin{bmatrix} 12 & 2 \\ -3 & 5 \end{bmatrix}$$

$$= 60-(-6) = 66$$